Université d'Ottawa · University of Ottawa
Faculté des sciences, Mathématiques et de statistique — Faculty of Science, Mathematics and Statistics

MAT 2377
Mi-term

Thursday June 16 2016 **Professor M. Alvo**

Time: 70 minutes
Student **Number:**________________________
Name: ________________________________

This is an open book test. Standard calculators are permitted**.** Answer all questions. **Place your answers in the table below and remit the entire exam.**

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Answer	B	D	A	A	E	C	B	D	D	A	C	D	B	A

1. A , B are two events such that $P(A) = .4$, $P(B) = .5$ and $P(A \cap B) = .3$ what is $P(A' \cup B')$?

   (A) $.03$ (B) $.7$ (C) 0 (D) 1 (E) $.5$
   $P(A' \cup B') = 1 - P(A \cap B) = 1 - .3 = .7$

2. Students on a boat have 9 flags to arrange on a pole. There are 3 red, 4 yellow and 2 blue flags. Flags of the same color are indistinguishable. How many different signals can be sent by arranging all the 9 flags on the pole?

   (A) 1009 (B) 4210 (C) 1360 (D) 1260
   (E) 2140
   $9!/(3!4!2!) = 1260$

3. A package of 24 tulip bulbs contains 8 yellow, 8 white and 8 blue bulbs. A second package of 24 tulip bulbs contains 6 yellow, 6 white and 12 blue bulbs. A package is chosen at random and the tulips are planted in a certain location. If 3 bulbs yielded 1 yellow 1 white and 1 blue tulip, what is the probability that the first package was selected?

   (A) $\frac{8^3}{8^3+(2)6^3}$ (B) $\frac{8^3}{8^3+6^3}$ (C) $\frac{6^3}{(2)8^3+6^3}$ (D) $\frac{6^3}{8^3+6^3}$ (E) $\frac{8^3}{(2)8^3+6^3}$
   This is a Bayes question. Each package has probability 0.5 of getting selected. if the first package is selected the probability of getting 1 yellow, 1 white and 1 blue is $8^3/\binom{24}{3}$. Similarly for the second package that probability is $[6(6)(12)]/\binom{24}{3}$. The desired probability is according to Bayes' theorem given by

$$\frac{\frac{1}{2}8^3/\binom{24}{3}}{\frac{1}{2}8^3/\binom{24}{3} + \frac{1}{2}[6(6)(12)]/\binom{24}{3}} = \frac{8^3}{8^3 + (2)6^3}$$

4. Let X be a discrete random variable having density $f(x) = cx, x = 1, 2, 3, 4$ for some constant c. Calculate the mean $E[X]$.
   (A) 3 (B) 10 (C) 30 (D) 2 (E) 2.5
   We must have $\sum_{x=1}^{4} f(x) = 1$.Hence, $c(1+2+3+4) = 10c$ and therefore $c = 1/10$ Now, $E[X]. = \frac{1}{10}(1+4+9+16) = 3$

5. I have 10 keys only one of which opens the door to my office. Every morning, I try one key after the other until I find the correct one. What is the probability that the correct key is the last one I try?

(A) $\frac{1}{10!}$ (B) 1 (C) $\frac{10}{10!}$ (D) $\frac{1}{5}$ (E) $\frac{1}{10}$
This was done in class and the answer is $1/10$

6. A boiler has 4 relief valves which operate independently. The probability that each opens properly is 0.99. What is the probability that at least one opens properly?
(A) $(.01)^4$ (B) $1-(.99)^4$ (C) $1-(.01)^4$ (D) $(.99)^4$ (E) 1
We compute the probability of the complimentary event. Hence the answer is $1-(.01)^4$

7. The length of time in minutes between consecutive calls to 911 in a small city has density

$$\begin{aligned} f_X(x) &= \frac{1}{20}e^{-x/20}, 0 < x < \infty \\ &= 0, otherwise. \end{aligned}$$

What is the probability that the time between consecutive calls is greater than 20 minutes?.

(A) $1/20$ (B) 0.368 (C) 0.632 (D) 0 (E) 1
Compute $\int_{20}^{\infty} \frac{1}{20}e^{-x/20}dx = e^{-1} = 0.368$

8. Flaws in a certain type of drapery material appear on the average of one in 150 square feet. Assuming a Poisson distribution, what is the probability of at most one flaw in 225 square feet?

(A) $\frac{1}{225}$ (B) $\frac{1}{150}$ (C) 0.442 (D) 0.558 (E) 1
This is a Poisson question. First compute $\mu = 225\left(\frac{1}{150}\right) = 1.5$. From the Poisson table, $P(X \leq 1) = 0.558$

9. A discrete random variable X has the following cumulative mass function:

$$F(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{20} & 0 \leq t < 1 \\ \frac{5}{20} & 1 \leq t < 2 \\ \frac{10}{20} & 2 \leq t < 3 \\ \frac{15}{20} & 3 \leq t < 4 \\ 1 & 4 \leq t \end{cases}$$

Calculate the conditional probability $P\left(X<3|X<4\right)$.

(A) $\frac{1}{20}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$.

$P\left(X<3|X<4\right)=\frac{P(X<3)}{P(X<4)}=\frac{10/20}{15/20}=2/3$

10. Let X,Y be independent random variables such that $E(X)=E\left(Y\right)=0$,$\sigma_X=\sigma_Y=5$.

Calculate $Var\left[\frac{(2X+3Y)}{5}\right]$.

(A) 13 (B) $\frac{13}{5}$ (C)5 (D)0 (E) 11

$Var\left[\frac{(2X+3Y)}{5}\right]=\frac{4\sigma^2+9\sigma^2}{25}=13$

11. The density of a discrete random variable X is given by

x	0	1	2	3	4
$f\left(x\right)$	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{4}{10}$	$\frac{1}{10}$	c

where c is a constant. Calculate $P\left(2.5\leq X\right)$

(A) $\frac{1}{10}$ (B) $\frac{2}{10}$ (C) $\frac{3}{10}$ (D) 0 (E) $\frac{4}{20}$

$3/10$

12. A candy maker produces mints whose weight follows a normal distribution with mean 21.37 gms and standard deviation 0.4 gm. Suppose 15 mints are selected at random. Let Y be the number of mints among them that weigh less than 20.857 gms. Find $P\left(Y\leq 2\right)$.

(A) 0.10 (B) 0.1841 (C) (D) 0.8159 (E) 0.2669

$p=P\left(X<20.857\right)=\Phi\left(-1.2825\right)=0.10$; $P\left(Y\leq 2\right)=0.8159$ from the binomial table

13. Let X,Y be the number of hand produced bicycles by two workers A, B respectively in a single day. The joint density is given below. Compute $P\left(X\geq Y\right)$.

x\y	0	1	2	3
0	0.00	0.05	0.10	0.10
1	0.05	0.10	0.10	0.10
2	0.10	0.10	0.10	0.10

(A) 0.30 (B) 0.45 (C) 0.55 (D) 0.20 (E) 0.35

$P(X \geq Y) = P(X = Y = 0) + P(X = 1, T = 0) + P(X = 2, Y = 0) + P(X = 1, Y = 1) + P(X = 2, Y = 1) + P(X + Y = 2) = 0.45$

14. Let X, Y be two random variables with joint density

$$f(x, y) = \begin{cases} 6y, & 0 < y < x < 1 \\ 0 & elsewhere \end{cases}$$

Calculate $P(Y < 0.5)$

(A) 0.5 (B) 0.10 (C) 0.15 (D) 0.20 (E) 0.25

We first calculate the marginal of Y. $f_Y(y) = \int_y^1 6y dx = 6y(1-y), 0 < y < 1$. Then

$$P(Y < 0.5) = \int_0^{0.5} f_Y(y)\, dy = \frac{1}{2}$$